Introduction to Cryptocurrency

Fall 2025

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November 4, 2025

Roadmap

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Hash Functions

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- Servers (to which you log in) store hashes, not passwords!
- "Chaotic" means deterministic but unpredictable: hash of a number never changes, but knowing hash of N doesn't reveal anything about hash of $N + 1$.
  - **One-way:** Can calculate outcomes, but can't back out initial conditions
  - **Collision Resistance:** Lots of outcomes are possible, so two initial conditions rarely result in same outcome (a "collision")

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A cryptographic hash function is a mapping $H : M \to D$ from a set M of possible messages of any length to a set D of digests, for example the set of strings of 0s and 1s of a fixed length, such as 256. See Ramzan (2012).

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In **digital signature schemes** (as we will see), a "digest" is a **signed message**.

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Collision Resistance and Invertability

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Answer: No, hash functions must be hard to invert.

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  - Bitcoins are mined by brute-forcing hashes to find xs that produce $H(x)$ with leading zeroes.

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  - But, using the public key, can't generate new valid signatures

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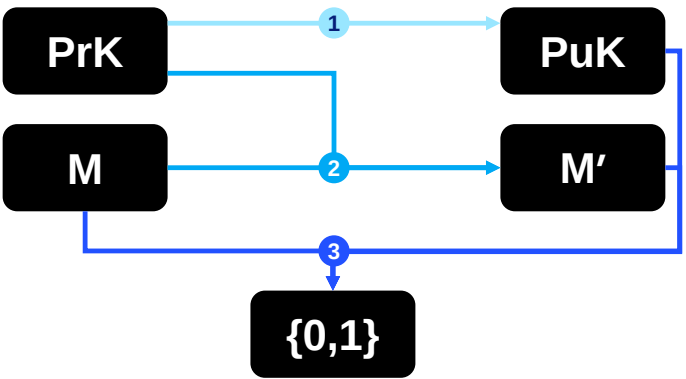
In Bitcoin, these "messages" encode a list of transfers of blocks from one owner to another!

Digital Signature Schemes, Visually



Ok, but how do these signatures actually work?

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Uses of Merkle Trees

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Merkle Trees are used in many applications to solve problems such as:

- Maintaining integrity of files stored on Dropbox.com, or *file outsourcing*
- Proving a transaction between Alice and Bob occurred, or *set membership*
- Transmitting files over unreliable channels, or *anti-entropy*

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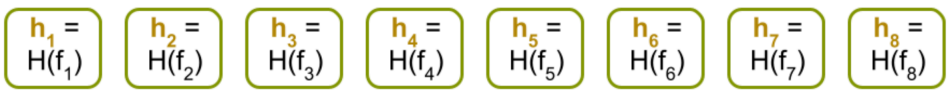
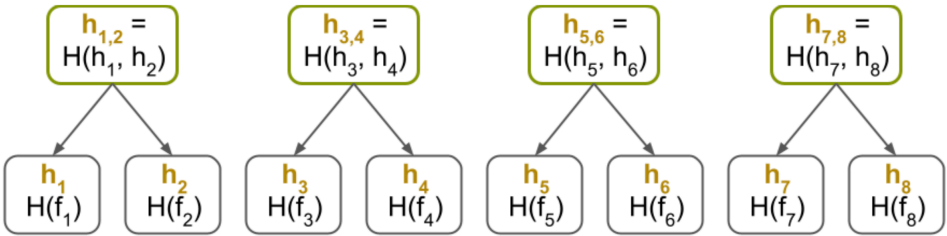
Start by hashing each file $H(f_i)$:



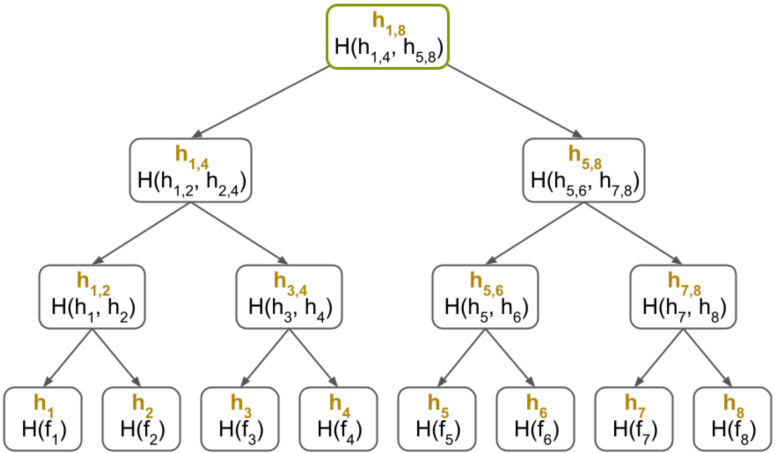
Construction of Merkle Trees

Then, create the second layer of the tree by hashing adjacent nodes together (this can be as simple as concatenating them!)



Construction of Merkle Trees

Continue until you have created an entire "tree":



Construction of Merkle Trees

Question: Why is this tree upside-down?

Construction of Merkle Trees

Question: Why is this tree upside-down?
Answer: Because Computer Scientists have never gone outside, a necessary prerequisite to see a real tree

Verification with Merkle Trees

Upload your files $(f_1 ... f_8)$ **and** your Merkle Tree to Dropbox, storing only the **Merkle Root** $h_{1,8}$ locally.

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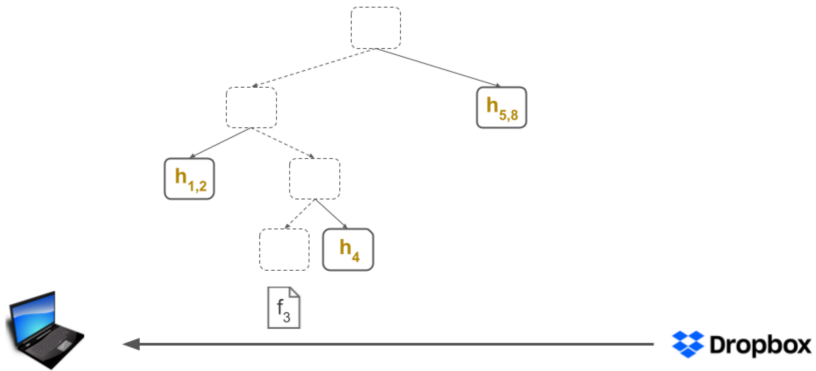
Later, ask for the file you want, say f_3, but make sure to also ask for the **Merkle Proof**: the hashes needed to compute $h_{1,8}$ from the desired **Merkle Leaf** (f_3)

Verification with Merkle Trees

The **Merkle Proof** lets you verify that you downloaded the real, original f_3:



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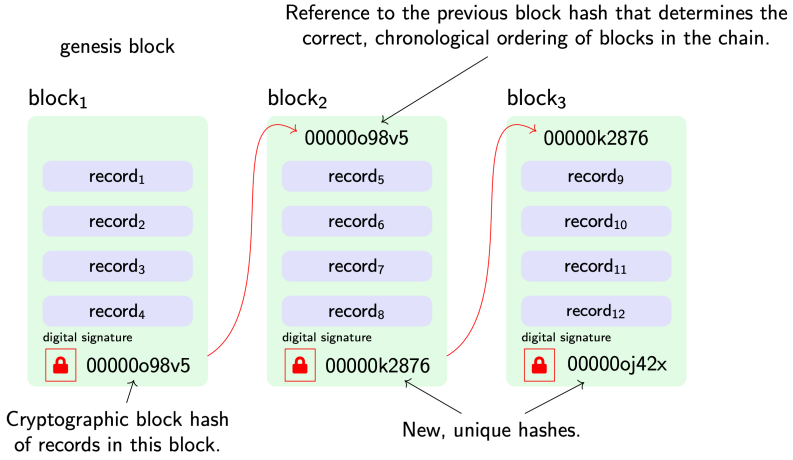
For other (analogous) uses of Merkle Trees including Transaction Verification and Transmitting files robustly over error-prone communication channels, see Tomescu (2020)

Roadmap

What is a Blockchain?

A **Blockchain** is a list of records that are linked together by hash functions



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  - No: the hashes of the last two messages would be wrong if the middle hash is missing.

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- Coinbase Base
- Ripple XRP
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Can we take away even more power from the admin? Can we make a set of rules to agree on a shared history of transactions without needing to trust anybody?

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- Bitcoin solves this through **proof of work**
- Ethereum solves this through **proof of stake**

Hash Puzzles

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  2. Proves that you did something hard to do
- With puzzle solution in hand, you become the admin!
- **Incentives:** solving the puzzle gives you some BTC as a block reward
- **Consensus:** everyone works on the longest chain

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- Can't just selectively delete the transaction from history (why?)
  - Hashes verify integrity of history!
- But...could go back in time, then create a parallel universe (a fork in the chain) where I never spent the money
  - This would be called a doublespend

Turning Back Time...

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- When would I succeed?
  - Because everyone uses the **longest chain**, I must be able to hash faster than **everyone else combined**
  - Control 51% of the computing power in the Bitcoin ecosystem!
- For more details, see Stanford CS251 slides

Roadmap

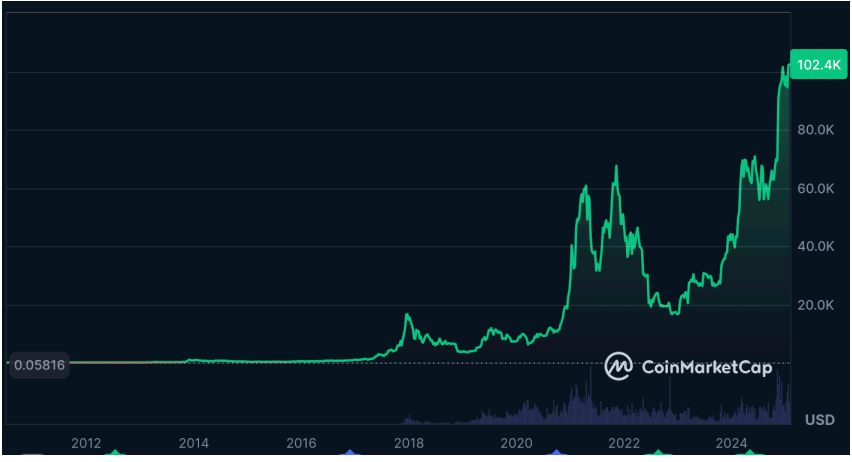
Satoshi Nakamoto

- Started sending emails about Bitcoin August 2008
- Oct 2008: Bitcoin white paper, code released
- January 9: first block mined!
  - "The Times 03/Jan/2009 Chancellor on brink of second bailout for banks"
- May 22, 2010: Laszlo Hanyecz bought two Papa John's pizzas for 10,000 BTC, now worth $600mil USD!
- Quickly became popular among black markets, Silk Road...
- Founder Ross Ulbricht was recently pardoned by Trump

Prizes for a Starcraft Tournament in 2011

Prizes:
1st Place: *$500*
2nd Place: *$250*
3rd Place: *$150*
4th Place: *$100*
5th-8th Place: *25 BitCoins*

Bitcoin Prices



Source: CoinMarketCap

Each BTC Block is a Merkle Tree of Transactions!

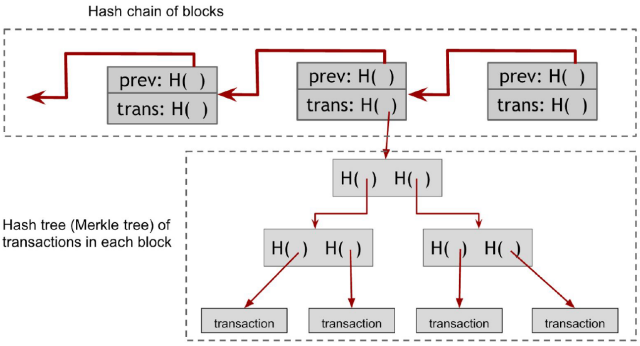


Figure 3.8. The Bitcoin block chain contains two different hash structures. The first is a hash chain of blocks that links the different blocks to one another. The second is internal to each block and is a Merkle Tree of transactions within the blocks.

- Valid block headers' hashes start with a big number of 0's (mining puzzle)

Source: NBFMG, fig. 3.8

Proof of Work Consensus

- Miners connected to each other in a network
- If you find a valid block, tell all other miners about it
  - They'll then mine the next block, based on yours
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- Why wouldn't I try to re-mine your block?
  - Consensus: if everyone else uses "longest chain", I do better by accepting your win, and trying to mine the next block

Proof of Work Consensus

- Notice that all properties depend on everyone "following the rules"...
  - If your block:
    - Doesn't hash to enough 0's, or
    - Steals $$, or
    - Includes invalid tx's, etc.
  - Others won't build on it, so others won't build on others not building on it,
  - So you won't get mining reward
- Simple rules are easy for a decentralized system to follow
  - By default, miners will pick the simplest possible path

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- What if I try to spend twice?
  - One tx is "invalid": doesn't obey the rules, since old tx output already spent
  - Remember, though; everything is ultimately due to consensus among miners, that "longest block of valid chains" is the legitimate blockchain

The Money Pot Game

- Two options:
  1. Get $1 for sure
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  1. Get $1 for sure
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- Which do you pick?

The Money Pot Game

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  - Everyone wants the money pot
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- "Equilibrium" behavior is easy to predict in such settings
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- Game theory language: call these congestion games, or wars of attrition, or tragedy of the commons

BTC mining

- Suppose mining a block pays 0.001 BTC
- Suppose mining block costs $60 in electricity
  - If BTC price $> \$60k$, everyone mines; blocks produced quickly
  - If BTC price $< \$60k$, nobody mines; tx's aren't included and the blockchain stops working!
- The fact that mining rate depends on BTC vs electricity price is not desirable...
- Solution: use difficulty adjustment to maintain constant block production rate

BTC Difficulty Adjustment

- Blocks should be produced every 10 minutes
- Every 2016 blocks (approx 14 days), Bitcoin considers average block production rate, and...
  - Increases/decreases hash problem difficulty, if block production is too fast/slow
- What determines how fast blocks are produced, fixing hash rate?

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- What determines how many people mine?
  - How profitable mining is, that is, how high BTC price is, vs electricity price

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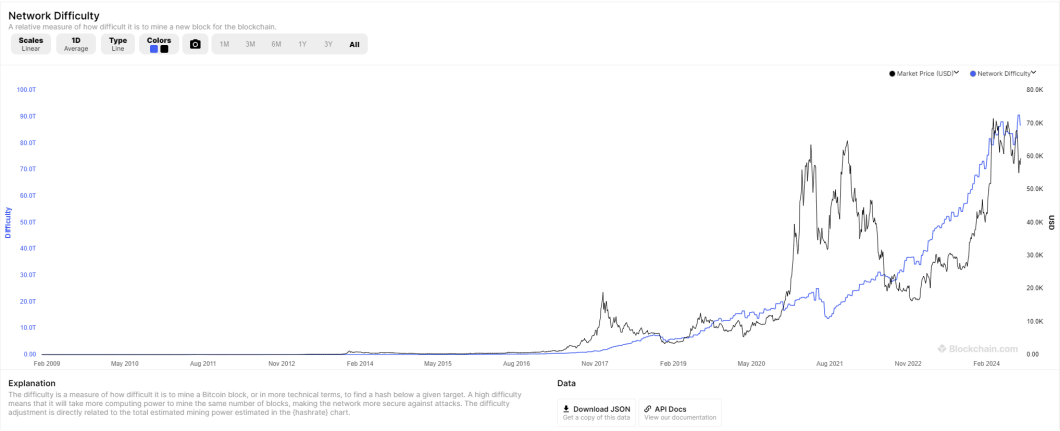
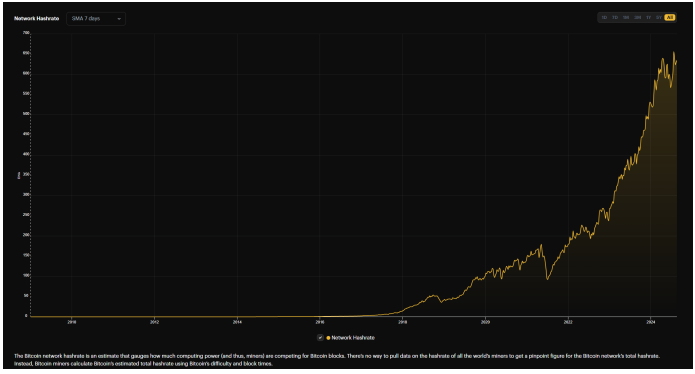
Prediction: difficulty should increase with BTC price

Difficulty vs BTC price



Source: Blockchain.com

Technological Improvement



- 842 Exahashes/s = 842 billion billion hashes a second!
- Each living person in the world could shake each other living person's hand 10 times, every second!
- $\sim 1\%$ of global energy consumption...

Source: Hashrate Index

BTC Halving

- Approximately every 4 years, issuance of BTC drops by half
- Last halving: April 2024, now 3.125 BTC per block
- Does this affect block production rate?
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  - No! Difficulty adjustment always ensure a constant block production rate.
- Also implies there will only ever be 21 million BTC in existence!
  - By approximately 2140, the block reward will drop to less than the smallest unit of bitcoin ($1/10^8$) and become equal to zero.
  - Difficulty adjustment ensures that block production rate is approximately constant

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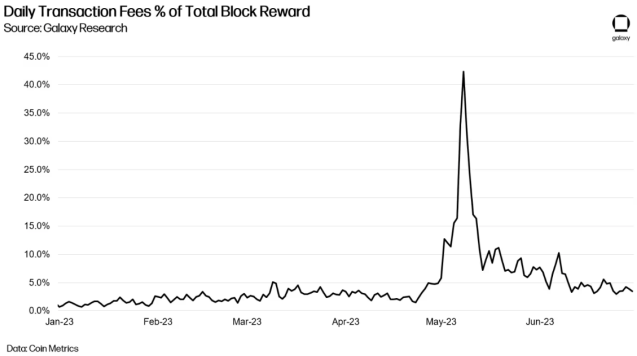
- A **"mempool"** is a pool of transactions waiting to be added to the bitcoin blockchain

Explore mempools at https://mempool.space/

Transaction Fees as % of Block Revenue



Source: Galaxy

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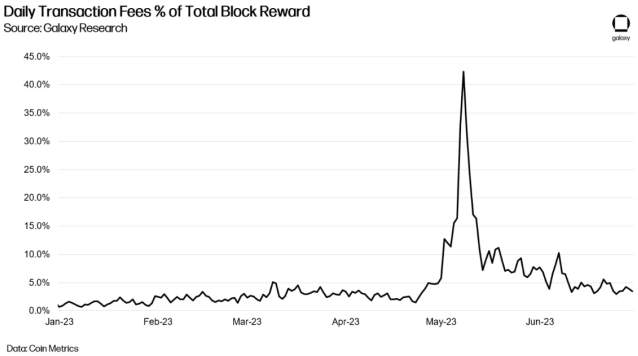

Source: Galaxy

In the long run, this will go to 100%! Why? Mining rewards will reach zero, and the only incentive to mine a block will be to collect transaction fees.

Incentives

- A centralized authority can "censor", i.e. not include tx's
- Decentralization and competition "ensure" performance
  - If you don't include my tx, the next miner will
  - Even if the US bans US miners from including tx's, someone else will include them

Further Reading on Mining Industry

- Galaxy digital 2023 and 2024
- Cool paper on mining pools

Roadmap

Smart Contracts

"A smart contract is essentially an automated agent that lives on the Ethereum network, has an Ethereum address and balance, and can send and receive transactions. A contract is "activated" every time someone sends a transaction to it, at which point it runs its code, perhaps modifying its internal state or even sending some transactions, and then shuts down."
-Ethereum Whitepaper, Vitalik Buterin (2014)

Smart Contracts

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  - A wallet, which can hold Ether (ETH),
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Deploying a smart contract or calling its functions requires a **gas fee**: ETH must be paid to the network to run the code.

Smart Contract Example: Tokens

- An ETH address, "technically", only "holds" Ether ("ETH") natively – no other tokens!
- An ETH address can be assigned non-ETH tokens through smart contracts
- A smart contract can define tokens using the **ERC-20** standard
  - So-called because it was originally the 20th submission to "Ethereum Requests for Comments"

Unlike BTC, ETH has native gas fees

"In Bitcoin, there are no mandatory transaction fees. Transactions can optionally include fees which are paid to miners, and it is up to the miners to decide what fees they are willing to accept. In Bitcoin, such a mechanism is already imperfect...In Ethereum, because of its Turing-completeness, a purely voluntary fee system would be catastrophic. Instead, Ethereum will have a system of mandatory fees, including a transaction fee and six fees for contract computations."
-Ethereum Whitepaper, Vitalik Buterin (2014)

Ethereum Virtual Machine Native Gas Costs

evm.codes

Proof of Stake

- Originally, ETH mining proof-of-work, similar to BTC
- On Sept 15 2022, ETH switched to proof of stake – called **"The Merge"**
- Reduced energy consumption by **over 99%**!
- Users deposit ("stake") 32 ETH into a special smart contract
- Stakers randomly selected (prop. to stake) to propose new blocks
- If a staker is caught trying to do "bad things" (e.g. propose 2 inconsistent blocks), they are "slashed": staked ETH is removed by smart contract
- Most newer blockchains (SOL, LUNA, AVAX) use proof-of-stake

Capacity of Ethereum

- Ethereum currently processes around 15 transactions per second
- This is pretty bad! Visa is 24,000 tps!
  - Every ETH staker has to run every transaction, and keep a copy of all data!
- Decentralization very expensive, in terms of computational efficiency!
- Proof-of-stake doesn't help throughput
  - Rollups do, which we will cover

Roadmap

Rollups or "L2's"

ETH is getting expensive! One set of solutions is rollups (also called L2's). Idea:

- Put our ETH, tokens, etc. in a "smart contract pot" (L2 bridge)
- People do transactions (smart contract, etc.) using funds in the pot, without doing anything on "main chain"
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- Need to guarantee whatever happens on "rollup" is exactly what would happen on ETH main chain! Two methods:
  - **Optimistic rollup** (Optimism, Arbitrum, Coinbase Base): Can "challenge" L2 transactions, will (essentially) run on L1 if challenged
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- Industry state:
  - $30B locked! ETH market cap is $408B, stables ~$141B
  - Optimistic rollups somewhat bigger than ZKs