Epstein-Zin: Why is $\psi > 1$?

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In short, the answer will be $\psi < 1$ implies that valuations are increasing in σ^2, which doesn't make any sense.

Consider a simple Lucas model, where consumption growth is

$$\Delta c_{t+1} = \mu + \sigma\epsilon_{t+1}$$

Wealth returns in Epstein-Zin are

$$r = -\log\beta + \frac{\mu}{\psi} - \frac{1}{2}\sigma^2\left(1-\psi^{-1}\right)(1-\gamma)$$

Deriving this is prohibitively difficult, but we trust Campbell. What we can do is build some intuition by showing how this reduces to the CRRA case when $\gamma\psi = 1$, the case in which EZ nests CRRA utility. With CRRA utility, the risk-free rate is

$$rf = -\log\beta + \gamma\mu - \frac{1}{2}\gamma^2\sigma^2$$

And the risk premium is

$$\gamma\sigma^2$$

Recall the relationship between total return and the risk premium

$$r_w - r_f + \frac{1}{2}\sigma^2 = rp = \gamma\sigma^2$$

So the total return in CRRA is

$$
\begin{aligned}
r_w &= r_f + rp - \frac{1}{2}\sigma^2 \\
&= -\log\beta + \gamma\mu - \frac{1}{2}\gamma^2\sigma^2 + \gamma\sigma^2 - \frac{1}{2}\sigma^2
\end{aligned}
$$

So recall that the total return from Epstein-Zin is

$$
r = -\log\beta + \frac{\mu}{\psi} - \frac{1}{2}\sigma^2\left(1-\psi^{-1}\right)(1-\gamma)
$$

If $\psi = \frac{1}{\gamma}$, then

$$
\begin{aligned}
r &= -\log\beta + \frac{\mu}{\psi} - \frac{1}{2}\sigma^2\left(1-\psi^{-1}\right)(1-\gamma) = -\log\beta + \frac{\mu}{\psi} - \frac{1}{2}\sigma^2(1-\gamma)^2 \\
&= -\log\beta + \frac{\mu}{\psi} - \frac{1}{2}\sigma^2\left(1-2\gamma+\gamma^2\right) \\
&= -\log\beta + \gamma\mu - \frac{1}{2}\sigma^2 + \gamma\sigma^2 - \frac{1}{2}\gamma^2\sigma^2
\end{aligned}
$$

Which is the same as in the CRRA case.

So what is the pd ratio? Campbell tells us that it is

$$
p - d = \sum \rho^j \Delta d - \sum \rho^j r
$$

Recall that in this type of model, equity is an asset that pays out aggregate consumption, and its dividends are aggregate consumption. So this is

$$
\begin{aligned}
p - d &= \sum \rho^j \Delta d - \sum \rho^j r \\
&= \frac{\mu}{1-\rho} - \frac{1}{1-\rho}\left(-\log\beta + \frac{\mu}{\psi} - \frac{1}{2}\sigma^2\left(1-\psi^{-1}\right)(1-\gamma)\right) \\
&= \frac{1}{1-\rho}\left(\mu + \log\beta - \frac{\mu}{\psi} + \frac{1}{2}\sigma^2\left(1-\psi^{-1}\right)(1-\gamma)\right) \\
&= \frac{1}{1-\rho}\left(\left(1-\psi^{-1}\right)\mu + \log\beta + \frac{1}{2}\sigma^2\left(1-\psi^{-1}\right)(1-\gamma)\right) \\
&= \frac{1}{1-\rho}\left(\left(1-\psi^{-1}\right)\left(\mu + \frac{1}{2}\sigma^2(1-\gamma)\right)\right)
\end{aligned}
$$

We believe that $\gamma > 1$, so the coefficient on σ^2 has the opposite sign from $\left(1-\psi^{-1}\right)$. $\psi < 1$ implies $\psi^{-1} > 1$, consequently $\left(1-\psi^{-1}\right)<0$, and the total coefficient on σ^2 is <u>positive.</u>

We have finally reached the punchline – $\psi < 1$ implies that price-dividend ratios are increasing in the volatility of consumption growth, which doesn't make any sense.